MATH419

Course Summary

Course : MATH419 **Title :** Set Theory
Length of Course : 8
Prerequisites : MATH305 **Credit Hours :** 3

Description

Course Description: This course continues the studies of set theory from MATH 210, Discrete Mathematics. This course expands coverage of logic and induction, sets, functions, recursive definitions, counting techniques and the Inclusion-exclusion principle. (Prerequisite: MATH305)

Course Scope:

This course is an interactive course designed to help students achieve a greater understanding of the set theory. The skills, tools and methodologies needed to analyze axiomatic systems and to judge the soundness of arguments and mathematical theorems are part of the purpose of this course. Axiomatic Set Theory is introduced and various implications of Zorn's lemma, Axiom of Choice and Continuum Hypothesis are discussed in order to give a broad exposure to a variety of current topics in this field. The course is designed for students majoring in mathematics with concentration in Applied Mathematics. Successful completion of this course will provide you with a working knowledge of the principles of set theory. A proper mastery of rigorous mathematical proofs is necessary in order to understand the proper application of the concepts introduced in our course.

Objectives

After completing the course, the student should be able to accomplish these Learning Objectives:

- CO-1 Use set notation and elementary set theory.
- CO-2 Describe the connection between set operations and logic.
- CO-3 Prove elementary results involving sets.
- CO-4 Assess Russell's paradox.
- CO-5 Construct short proofs using direct proof, indirect proof, proof by contradiction, and case analysis.
- CO-6 Construct elementary proofs using ordinary and strong induction in the context of studying the properties of recursion, relations, and graph theory.

Outline

Week 1: Getting Started & Becoming familiar with the subject matter of Set Theory and the background material in Mathematical Logic

Learning Objective(s)

LO-1
be able to formalize mathematical statements, use logical analysis and apply rules of induction and recursion both in finite and transfinite modes; mastering logical analysis and understand both the power (validity) and limitation of formal systems;

Reading(s)

Read Week 01 Announcement and follow its directions;

Read the Syllabus carefully;

Go over Lesson 01 in Sakai;

Read and study Chapters 01 from Tourlakis book;

Print out and browse the supplementary material on the Foundations.

Assignment(s)

First required contact. Introduce yourself on the Week 01 Introduction Forum and get to know your classmates

Review Problems:
Chapter 01 Review (not-graded):

Complete:
Work on Weekly **Test 01**; Access Test 02 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 2: Naïve Set Theory; basic notions and terminology;

Learning Objective(s)

LO-2
master the basic concepts and notation of in naive and intuitive set theory; be familiar with various paradoxes in intuitive and naive set theory and understand the need for formalization and axiomatization of set theory;

Reading(s)

Read Week 02 Announcement and follow its directions;

Read and study Chapter 02 and Lesson 02 in Sakai;

Assignment(s)

Review Problems:
Chapter 01 Review (not-graded):

Complete:
Participate in Week 02 Forum;

Work on Weekly Test 02; Access Test 02 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 3: Axiomatic Set Theory; Paradoxes, Sets and Classes;

Learning Objective(s)

LO-3
be familiar with different axiomatic systems of set theory such as Neumann-Bernays-Gödel and Zermelo-Fraenkel systems;

Reading(s)

Read Week 03 Announcement and follow its directions;

Read and study Chapter 03 and Lesson 03 in Sakai;

Participate in Week 03 Forum;

Assignment(s)

Review Problems:
Chapter 03 Review (not-graded):

Complete:
Participate in Week 03 Forum;

Work on Weekly Test 03; Access Test 03 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 4: Axiom of Choice, Zorn's lemma, Continuum Hypothesis;

Learning Objective(s)

LO-4, LO-5
Use the set theoretic approach to formalize the concepts of mapping, relations and functions; be able to apply the concepts of relations and functions and mapping in various mathematical disciplines; be familiar with variants of the Zorn's lemma, Axiom of Choice and Continuum Hypothesis;be able to carry out proofs and constructions by transfinite induction and recursion;.

Reading(s)

Read Week 04 Announcement and follow its directions;

Read and study Chapter 04 and Lesson 04 in Sakai;

Participate in Week 04 Forum;

Assignment(s)

Review Problems:

Chapter 04 Review (not-graded):

Complete:
Participate in Week 04 Forum;

Work on Weekly Test 04; Access Test 04 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 5: Natural Number Theory

Learning Objective(s)

LO-5
be familiar with the theory of natural numbers based on set theoretic concepts; be able to carry out proofs and constructions by transfinite induction and recursion; master the algebra of relations and functions; be familiar with equivalence relations;

Reading(s)

Read Week 05 Announcement and follow its directions;

Read and study Chapter 05 and Lesson 05 in Sakai;

Participate in Week 05 Forum;

Assignment(s)

Review Problems:
Chapter 05 Review (not-graded):

Complete:
Participate in Week 05 Forum;

Work on Weekly Test 05; Access Test 05 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 6: Ordinal and Cardinal Arithmetic

Learning Objective(s)

LO-6
master counting techniques and ordinal and cardinal arithmetic; master the concepts of well-ordering and partial ordering;

Reading(s)

Read Week 06 Announcement and follow its directions;

Read and study Chapter 06 and Lesson 04 in Sakai;

Participate in Week 06 Forum;

Assignment(s)

Review Problems:
Chapter 06 Review (not-graded):

Complete:
Participate in Week 06 Forum;

Work on Weekly Test 06; Access Test 06 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Post questions or comments on the Q&A Forum.

Week 7: Application to formal language theory;

Learning Objective(s)

LO-7
be familiar with applications of logic and set theory to modeling computation, applications in formal language theories;

Reading(s)

Read Week 07 Announcement and follow its directions;

Read and study Chapter 07 and Lesson 04 in Sakai;

Participate in Week 07 Forum;

Assignment(s)

Review Problems:
Chapter 07 Review (not-graded):

Complete:
Participate in Week 07 Forum;

Finish the Writing Project; by 11:59 PM Eastern Time on Sunday. Access Test 07 in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once.

Review course material and get ready for the final test.

Post questions or comments on the Q&A Forum.

Week 8: Inclusion-Exclusion Principle;

Learning Objective(s)

LO-8
be conversant in solving linear recurrence relations and their usage in various mathematical disciplines; master the inclusion-exclusion concepts and apply them.

Reading(s)

Read Week 08 Announcement and follow its directions;

Read and study Chapter 08 and Lesson 08 in Sakai;

Participate in Final Week Forum;

Finish preparation for taking the Final Test;

Assignment(s)

Review Problems:
All the homework during the course (not-graded):

Participate in Final Reflections Forum;

Finish the Final Test; by 11:59 PM Eastern Time on Sunday. Access the Final Test in the Tests & Quizzes section of our classroom. Remember you will be able to submit your answers only once. Course Ends

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Evaluation

Staying on task and adhering to the published schedule are typically among the most challenging aspects of completing an academic course successfully. This is especially true for online learning environment. To avoid the pitfall of falling behind, students in this course should complete the assigned reading and review the PowerPoint presentations which can be linked via the Lessons section of the online classroom in the Learning Management System, Sakai, in a timely manner. Students should also complete the suggested Review Problem Sets as set forth in the schedule provided in the Course Outline of this syllabus. Review Problem Sets will not be graded, but their mastery is crucial to a proper understanding of the topics covered in this course. Students should review these problems as a means to confirm their understanding of the topics covered in the textbook.

I urge you to utilize the Q&A Forum as a means to interact with your classmates. If while working through examples or problems from our textbook you have a question or a comment, please post the question or comment on the Q&A Forum. Naturally, I hope that the Q&A Forum will facilitate interactions among the members of our class. If you have an insight that you wish to share or if you have a question please use the Q&A Forum to exchange such information.

Student grades for the course will be based on class and **forum participation, a writing project, six weekly tests**, and the **Final Test**.

<u>**Forum and Class Participation**</u>: Naturally, I value punctuality, familiarity with the required readings, and classroom questions or comments that are relevant and insightful. Whether helping someone understand a point, seeking clarification of a concept you may not completely understand, or contributing to the positive flow of the class discussion based on your experience, it is important for you to realize that learning is an action process—and sharing is a key ingredient in undertaking that process successfully. Therefore, I urge you to participate actively in an effort to build a positive and effective learning environment--for yourself and others.

Class participation via the Discussion Forums during weeks 1, 2, 3, 4, 5, 6, 7 and 8 is a course requirement and will count as 12 percent of the overall course grade. You are required to respond to the questions posed in these Discussion Forums by making an initial post in the Discussion Forum by 11:59PM EASTERN time on Wednesday during the week in which a discussion question is posed. I will evaluate your responses to each of these questions using a 0 to 10 scale, and your contribution to each of these six Discussion Forums will count as 2 percent of the overall course grade, for a total of 12 percent. Keep in mind that you need not necessarily answer a discussion question correctly to earn full credit for your post. My evaluation will be based on the extent to which you participated and fostered a positive and effective learning environment--for yourself and others. Participating and sharing are the keys. Collectively, I'm confident that we will derive the correct response to each of these discussion questions. To make a post to a Discussion Forum, click on the Forum topic link, and then click Post New Thread. In the title block of the dialog box that appears kindly insert your first and last name; compose your post in the message box; and then click Post Message. I will post the answers to these discussion questions by making a post myself after the Wednesday 11:59PM Eastern Time deadline. At that time the Discussion Forum will be locked and no additional posts will be permitted.

Naturally, I urge you to read my solution post, the posts of your classmates, and the feedback I provide. If you wish to continue to discuss a the question posed in a forum that has been locked, you can certainly do so by using the Message tool or the Q&A Forum to interact with the other members of our course.

The Week 1 Introduction Forum: During the first week of class each student must make a post to the Week 1 Introduction Forum. You are to use this Forum to introduce yourself and state your goals and objectives as they relate to our course. You are required to make a post to the Week 1 Introduction Forum in order to complete your enrollment in the course. Your post must be at least 250 words, and you must complete it by the end of the first week. This is a university requirement. To make a post to the Week 1 Introduction Forum, click on the Forum topic link, and then click Post New Thread. In the title block of the dialog box that appears kindly insert your first and last name; compose your post in the message box; and then click Post Message.

Besides completing your enrollment in the course, the Week 1 Introduction Forum is designed to 1) build peer-to-peer relationships by introducing oneself and one's background to the class; 2) to articulate individual student learning goals and/or expectations for the class. Therefore, in your introduction you may wish to touch upon the following:

1. Who you are and how you would like to be addressed.

2. Your academic major or program of study.

3. Your current status in your program of study.

4. Your academic goals including why you are taking this course and what you hope to achieve by completing it.

5. Other information about yourself that you would like to share.

6. Present your favorite mathematical equation within the forum.

Naturally, I will review every post made to the Week 1 Introduction Forum and award 2 percentage points of extra credit to every student making a post that promotes the aim of building peer-to-peer relationships and articulates one's learning goals and aspirations with respect to our course. This and the last week Conclusion Forum will be the only extra credit opportunities provided in our course.

Tests and Quizzes: Six weekly tests and a final comprehensive test will be utilized to evaluate your performance in the course. Each weekly test will account for 12% of your overall grade. Final test will also account for 12% of your overall grade. Generally, the tests will contain problems similar to those discussed in the suggested homework problems and the many examples given in our textbook. However, you should expect to be challenged by the graded exercises. Tests will be conducted as indicated on the course schedule and students are expected to complete them on time. No late submissions will be accepted. Specific instructions will be provided for each test in the Announcements section of our classroom at the outset of the week in which these graded exercises are due. Each of these graded assessments is to be completed on an individual basis. You may consult published textbooks, articles, and other printed materials. However, **no collaboration is permitted on the examinations or tests.** You are not to discuss, orally, in print—in any manner—any aspect of the graded exercises with anyone other than your instructor. Clearly, student-teacher relationships are built on trust. This is especially true in the case of an online course. For example, students must trust that teachers have made appropriate decisions about the structure and content of the courses they teach, and teachers must trust that students complete assignments as directed. Acts that violate this trust undermine the educational process and compromise the integrity of the perpetrator. Don't cheat. Don't compromise your integrity. To do so invalidates the very purpose which likely motivated you to undertake this course—to learn, to become a better decision maker, to broaden your perspective, and to increase your skill set.

At the beginning of the week in which they are due, tests will be posted in the **Tests & Quizzes** section of our classroom. When you are prepared to take an assessment go to the Tests & Quizzes section of our classroom and click on the assessment. It is important for you to understand that you will be able to submit

your answers to an assessment only once. Your answers must be submitted by the 11:59PM Eastern Time deadline, as indicated in the syllabus and the course calendar. I will not accept late submissions. So, please don't wait until the last minute to submit your answers to an assessment or test. As soon as you submit your answers your assessment will be graded, and your score will be recorded in the Gradebook. The correct answers will be available 24 hours after the deadline and once everyone has submitted their answers. At that time, you can access the feedback by clicking on the assessment in the **Tests & Quizzes** section of our classroom. Naturally, if you answer any of the questions on an assessment incorrectly I urge you to review the feedback and reconcile any errors you may have made on a quiz or exam.

The notations used in set theory aren't found in many word processing programs, making it difficult to produce many of the symbols used in our course. You may wish to use the Symbol font in Microsoft Word and the Insert/Object/Microsoft Equation feature in Word when preparing documents related to our course. Insert/Symbol is also sometimes useful. Of course, you will also want to familiarize yourself with the Insert/Edit Equation feature (the fx icon) contained in the Rich Text Editor that is available in the Forums section of our classroom. Additionally, since many of the computations and analyses required in our course can be easily carried out using Microsoft Excel, you may wish to familiarize yourself with the process whereby Excel outputs can be copied and pasted into a Word or pdf file.

Forum Assignments: There will be an Introductory Forum and a Final Conclusion Forum each worth 2% of the final grade (as extra credit) and 6 other Forums in Weeks 2, 3, 4, 5, 6 and 7 worth 2% each for a total of 12% of the final grade.

Tests: There will be 6 weekly tests in the "Tests and Quizzes" section of the Course for weeks 01 through 06, each worth 12% for a total of 72% of the final grade. **Usually, you have two weeks to work on a Test.**

Final Test: There will be a Final Test worth 12% of the Final Grade. It is recommended that you review all the tests and the study material before trying to take the final test.

*** Homework Assignments: There will be weekly suggested homework problems during each week of instruction. These problems are for practice and are not graded, yet they are an important and integral part of this class and you are strongly encouraged to work on them on a regular basis.***

Students' final grades will be posted within 7 days of the end of the semester. Students should not telephone the university looking for grades until at least 30 days after the end of the semester.

Please see the Student Handbook to reference the University's grading scale.

Writing Project Instructions:

This project is due at the end of week 07 of the course.

In this course, we have used the weekly forum discussions to delve into Set Theory's applications, asking such questions as: How is Set Theory used in the different branches of mathematics? Is it a common or uncommon technique? Is it used to cut complexities of arguments? Increase the transparency of proofs? Solve problems that are not otherwise easily (or time effectively) solvable? Unfortunately, the weekly forum discussions by their nature have provided only limited opportunity to learn about how Set Theory is used in different branches of mathematical sciences.

With that in mind, for this writing project we are going to delve in more depth into this issue.

Please choose one Set Theory related topic. You can choose any of the topics we've covered in this course, or even a topic that we did not cover. As long as it relates to Set Theory, it is fine. For a quick list of the topics we've covered, just look at the titles of the subsections of each chapter in the textbook or supplementary reading material provided throughout the course.

Then pair your chosen topic with an example from various branches of mathematical sciences of how that topic can be applied to solve problems. Examples include (but are by no means limited to):

- Theory of Syntactic Structures and Languages

- Computability of Undecidability
- Proof Theory
- Graphs
- Trees
- Model Theory

There are numerous examples of Set Theory applications scattered throughout our textbook: in the first introductory pages of each chapter, in selected chapter sections (e.g., 1.10), and contained within the description of many homework problems located throughout the chapter. You can also find examples at any of the websites mentioned in the weekly forum descriptions, or at websites you research and find yourself.

Once you have picked a topic and application, you are ready to do research and write up your findings. For your chosen "Topic + Application" combination, be ready to answer the following questions:

Describe the Set Theoretic topic in general. Give a brief overview of the mathematics involved. If you are writing about (for example) finite versus infinite axiomatizations, then your topic is comparisons of different axiomatizations of Set Theory and you would write an overview of it, much as you would find in a textbook.

Describe the full range of applications for your chosen topic. Although the rest of your paper will be devoted to the single application of your chosen topic, for this section the idea is to paint a picture of the full range of applications that are possible. For example, logic and set theory can be used anywhere that you have inductive and recursive reasoning, and this occurs not only in abstract mathematics but also in linguistics, theory of computation, automata, and the like.

Be sure to list each of the different application areas and briefly (a sentence or two for each different application) indicate how your chosen topic is used for each type of application. For example, in linguistics is used to model how human beings acquire speech and are able to communicate and make conventions or agreements.

Now it is time to zero in on the specific "topic + application" combination that you have chosen. In this part of the paper, you should describe how the specific topic you chose is applied to the area you have chosen. Essentially this is the same idea as item 2 above, but because you are focusing here on only your one single chosen application, you can go into it in much more depth and detail than you did for the brief overviews you wrote in item 2.

Continuing with our example, you might describe in detail how Set Theory is used within linguistics. What kinds of fundamental issues regarding human mind and intelligence is it used for? Could the linguistic problems be addressed any other way, and if so what is the advantage of using set theoretic concepts? To what extent, if any, is the process computerized and what changes, if any, are made to it in order to accommodate computerization? These particular questions might not apply to your own particular topic+application, but I am sure you can think of many similar aspects to discuss once you have done the research.

The last part of the paper centers on doing some illustrative conclusions.

The total paper should be approximately 3-5 pages in length (not counting the cover page, references page, or any exhibits), and include a full set of references (including the textbook if you use it, as I assume you will). APA format requires that the font used should be Arial or Times New Roman, 10 or 12 point only; the pages be double spaced; and the margins should be no more than one inch all around.

Be sure to fully reference your paper, including all references you used (textbook, other books and all websites).

Please format the paper in APA style. You can find an online APA style guide in the APUS Library. Navigate to APUS Log in » Online Library: CampusGuides » Library Tutorial Center » Writing Help or you can use the link http://apus.campusguides.com/writing/citation and then click on APUS APA Style Guide. Another excellent APA resource is found by googling "The Owl at Purdue" (OWL is the online writing center at Purdue University) – there you will find not only a style guide but a sample paper that you can use as a model for what your paper should look like.

Submission Instructions:

Please post your completed papers in the Assignments area (Assignments tab on the left hand side of the Sakai classroom). Your assignment is automatically submitted to TurnItIn with your submission.

Be sure that you attach your assignment in the Assignment area, attached as a file attachment in doc or docx format (Word format). Do not paste or write your assignment into the text box, as this cannot be run through TurnItIn (which is required).

Note that no work is accepted once the course is over, so for all practical purposes, you cannot submit this assignment late. Be sure to plan your work and your time accordingly.

The final grade in the course will be based on the writing project, six weekly tests, final test and six forums, as indicated below. Grades will be assigned based on the following scores:

Grading:

Name	Grade %
Forums	12.00 %
Forum Week 02	2.00 %
Forum Week 03	2.00 %
Forum Week 04	2.00 %
Forum Week 05	2.00 %
Forum Week 06	2.00 %
Forum Week 07	2.00 %
Tests	84.00 %
Test-01	12.00 %
Test-02	12.00 %
Test-03	12.00 %
Test-04	12.00 %
Test-05	12.00 %
Test-06	12.00 %
Final-Test	12.00 %
Writing Project	4.00 %
Writing Project	4.00 %
Extra Credit	0.00 %
APUS Honor Code and Pledge	0.00 %
Introduction Forum Week 01	0.00 %
Conclusion Forum	0.00 %

Materials

Book Title: Lectures in Logic and Set Theory, Volume 2: Set Theory - e-book available in the APUS Online Library

Author: Tourlakis

Publication Info: Cambridge University Press

ISBN: 9780521753746

Book Title: To find the library e-book(s) req'd for your course, please visit http://apus.libguides.com/er.php to locate the eReserve by course #. You must be logged in to eCampus first to access the links.

Author: N/A

Publication Info: N/A

ISBN: N/A

Lectures in Logic and Set Theory, Volume 2: Set Theory - e-book available in the APUS Online Library. Online Access: http://site.ebrary.com/lib/apus/Doc?id=10070006

This course will use Microsoft Office for some of the work. Students should have a basic familiarity with Microsoft Office and have access to this software application.

In addition to the required course texts, there are many public domain Websites that are useful. Please abide by the university's academic honesty policy when using Internet sources.

Course Guidelines

Citation and Reference Style

- Attention Please: Students will follow the APA Format as the sole citation and reference style used in written work submitted as part of coursework to the University. Assignments completed in a narrative essay or composition format must follow the citation style cited in the APA Format.

Tutoring

- Tutor.com offers online homework help and learning resources by connecting students to certified tutors for one-on-one help. AMU and APU students are eligible for 10 free hours* of tutoring provided by APUS. Tutors are available 24/7 unless otherwise noted. Tutor.com also has a SkillCenter Resource Library offering educational resources, worksheets, videos, websites and career help. Accessing these resources does not count against tutoring hours and is also available 24/7. Please visit the APUS Library and search for 'Tutor' to create an account.

Late Assignments

- Students are expected to submit classroom assignments by the posted due date and to complete the course according to the published class schedule. The due date for each assignment is listed under each Assignment.
- Generally speaking, late work may result in a deduction up to 15% of the grade for each day late, not to exceed 5 days.
- As a working adult I know your time is limited and often out of your control. Faculty may be more flexible if they know ahead of time of any potential late assignments.

Turn It In

- Faculty may require assignments be submitted to Turnitin.com. Turnitin.com will analyze a paper and report instances of potential plagiarism for the student to edit before submitting it for a grade. In some cases professors may require students to use Turnitin.com. This is automatically processed through the Assignments area of the course.

Academic Dishonesty

- Academic Dishonesty incorporates more than plagiarism, which is using the work of others without citation. Academic dishonesty includes any use of content purchased or retrieved from web services such as CourseHero.com. Additionally, allowing your work to be placed on such web services is academic dishonesty, as it is enabling the dishonesty of others. The copy and pasting of content from

any web page, without citation as a direct quote, is academic dishonesty. When in doubt, do not copy/paste, and always cite.

Submission Guidelines

- Some assignments may have very specific requirements for formatting (such as font, margins, etc) and submission file type (such as .docx, .pdf, etc) See the assignment instructions for details. In general, standard file types such as those associated with Microsoft Office are preferred, unless otherwise specified.

Disclaimer Statement

- Course content may vary from the outline to meet the needs of this particular group.

Communicating on the Forum

- Forums are the heart of the interaction in this course. The more engaged and lively the exchanges, the more interesting and fun the course will be. Only substantive comments will receive credit. Although there is a final posting time after which the instructor will grade comments, it is not sufficient to wait until the last day to contribute your comments/questions on the forum. The purpose of the forums is to actively participate in an on-going discussion about the assigned content.
- "Substantive" means comments that contribute something new and hopefully important to the discussion. Thus a message that simply says "I agree" is not substantive. A substantive comment contributes a new idea or perspective, a good follow-up question to a point made, offers a response to a question, provides an example or illustration of a key point, points out an inconsistency in an argument, etc.
- As a class, if we run into conflicting view points, we must respect each individual's own opinion. Hateful and hurtful comments towards other individuals, students, groups, peoples, and/or societies will not be tolerated.

University Policies

Student Handbook

- Drop/Withdrawal policy
- Extension Requests
- Academic Probation
- Appeals
- Disability Accommodations

The mission of American Public University System is to provide high quality higher education with emphasis on educating the nation's military and public service communities by offering respected, relevant, accessible, affordable, and student-focused online programs that prepare students for service and leadership in a diverse, global society.